Mark Scheme (Results)

January 2020

Pearson Edexcel International GCE

in Pure Mathematics P3 (WMA13) Paper 01

Question Number	Scheme	Marks
1 (a)	$P_0 = 300$	B1 (1)
(b)	$420 = \dfrac{900e^{0.12t}}{2e^{0.12t}+1} \Rightarrow 60e^{0.12t} = 420$	M1 A1
	Correct use of lns $\Rightarrow t = \dfrac{\ln 7}{0.12} = 16.22$	dM1 A1 (4)
(c)	States that maximum number (upper limit) is 450 so cannot reach 500	B1 (1)
		6 marks

(a)

B1 300

(b)

M1 Substitutes $N = 420$ and proceeds to $Ae^{0.12t} = B$ condoning slips

A1 $60e^{0.12t} = 420$ oe

dM1 Uses correct ln work to find *t*. This must be from a solvable equation.

Method 1: $Ae^{0.12t} = B \rightarrow e^{0.12t} = k \rightarrow 0.12t = \ln k \rightarrow t = ...$ $(k > 0)$

Method 2: $Ae^{0.12t} = B \rightarrow \ln A + 0.12t = \ln B \rightarrow t = ...$ $(A, B > 0)$

A1 Awrt 16.22 (years)

Note: **Answers without working** (even to accuracy of 1 dp) can score SC 1100.

Eg. $420 = \dfrac{900e^{0.12t}}{2e^{0.12t}+1} \Rightarrow t = 16.2$

(c)

B1 May be tackled in a variety of ways. **Requires a reason and a conclusion**

E.g.

- States that the **upper** limit (or maximum value) is 450 so cannot reach 500. Note that it is acceptable to state that the number of toads cannot exceed 449. (Allow here $N < 450$ for upper limit is 450)
- The conclusion can be implied by a statement such as "as the maximum value is 450"
- Alternatively substitutes $500 = \frac{900\mathrm{e}^{0.12t}}{2\mathrm{e}^{0.12t}+1} \Rightarrow 100\mathrm{e}^{0.12t} = -500$ or similar and states this cannot be found as ln's cannot be taken of negative numbers, hence cannot ever be 500. (Allow the candidate to state maths error, exponentials cannot be negative to form part of their reason)

The calculations here must be correct and the proof complete.

The following 3 examples score B0 as they are incomplete and/or incorrect

1) $500 = \frac{900\mathrm{e}^{0.12t}}{2\mathrm{e}^{0.12t}+1} \Rightarrow \mathrm{e}^{0.12t} = -5$ which cannot be found. (Requires a conclusion)

2) The limit is 450 so they cannot reach 500. (Requires upper limit)

3) $500 = \frac{900\mathrm{e}^{0.12t}}{2\mathrm{e}^{0.12t}+1} \Rightarrow \mathrm{e}^{0.12t} = -0.01$ which cannot be found and so they cannot reach 500.

(Requires correct calculation)

Question Number	Scheme	Marks
2.(a)	$\mathrm{fg}\left(\mathrm{e}^2\right)=\mathrm{f}\left(\frac{5}{2}\ln\mathrm{e}^2\right)=\frac{12}{\frac{5}{2}\ln\mathrm{e}^2+1},=2$	M1, A1
		(2)
(b)	$\mathrm{f}(x)=\frac{12}{x+1}$	
	$\mathrm{f}^{-1}(x)=\frac{12}{x}-1$	M1 A1
	$0<x<12$	B1
		(3)
(c)	$\frac{12}{x+1}=\frac{12}{x}-1\Rightarrow 12x=12\left(x+1\right)-x(x+1)$	M1
	$\Rightarrow x^2+x-12=0\Rightarrow x=...$ Must be 3TQ	dM1
	$x=3$ only	A1
		(3) **8 marks**
(c) Alts	Solves $\mathrm{f}^{-1}(x)=x\Rightarrow\frac{12}{x}-1=x$ leading to quadratic equation, or solves $\mathrm{f}(x)=x\Rightarrow\frac{12}{x+1}=x$ leading to quadratic equation	M1
	$\Rightarrow x^2+x-12=0\Rightarrow x=...$ Must be 3TQ	dM1
	$x=3$ only	A1
		(3)

(a)

M1 Correct order of operations applying g before f on e^2.

Allow an attempt to substitute $x=\mathrm{e}^2$ into $\mathrm{fg}\left(x\right)=\frac{12}{\frac{5}{2}\ln x+1}$

This must be a complete attempt using all aspects of both functions but allow for slips on the coefficients

A1 cso 2.

(b)

M1 Changes the subject of $y=\frac{12}{x+1}$ or $x=\frac{12}{y+1}$ to one of the appropriate forms. See below.

Allow for $x=\frac{12}{y}\pm 1$ or $x=\frac{12\pm y}{y}$ (Only error allowed is a slip in sign)

Or $y=\frac{12}{x}\pm 1$ or $y=\frac{12\pm x}{x}$ when x and y are swapped

A1 $\mathrm{f}^{-1}(x)=\frac{12}{x}-1$ or $\mathrm{f}^{-1}(x)=\frac{12-x}{x}$ **with correct notation**. Condone $\mathrm{f}^{-1}(x)=y=\frac{12}{x}-1$

$\mathrm{f}^{-1}:x\mapsto\frac{12}{x}-1$ is fine as is any other variable used consistently, e.g. $\mathrm{f}^{-1}(y)=\frac{12}{y}-1$

but $\mathrm{f}^{-1}=\frac{12}{x}-1$ is A0 (as it is incomplete) and $y=\frac{12}{x}-1$ is also A0 (not set in the appropriate form)

B1 Gives a correct domain for the function $0<x<12$ or equivalent "Domain $\in\left(0,12\right)$"

(c)

M1 Attempts to set $\mathrm{f}^{-1}(x)=\mathrm{f}(x)$, $\mathrm{f}^{-1}(x)=x$ $\mathrm{ff}(x)=x$ or $\mathrm{f}(x)=x$ and proceeds to a quadratic equation

The quadratic equation does not need to be simplified. Allow this mark for using **their** $\mathrm{f}^{-1}(x)$

dM1 Solves a 3TQ leading to at least one value for x. Apply the usual rules.
This is dependent upon the previous M mark

A1 $x=3$ only

Question Number	Scheme	Marks
3.(a)	Implies equation of line is of the form $\log_{10} y = \pm\frac{2}{3}\log_{10} x + 4$	M1
	States $\log_{10} y = -\frac{2}{3}\log_{10} x + 4$ o.e.	A1
		(2)
(b)	Applies one correct log law E.g $\log_{10} y = -\frac{2}{3}\log_{10} x + 4 \rightarrow \log_{10} y = \log_{10} x^{-\frac{2}{3}} + 4$	M1
	Full attempt to undo the logs $\log_{10} y = \log_{10} x^{-\frac{2}{3}} + \log_{10} 10^4 \rightarrow y = x^{-\frac{2}{3}} \times 10^4$	dM1
	$\rightarrow y = 10\,000x^{-\frac{2}{3}}$ o.e.	A1
		(3) **5 marks**

Note that other versions of $\log_{10}$ are acceptable. So allow $\log_{10} \leftrightarrow \log$ and $\log_{10} \leftrightarrow \lg$

(a)

M1 Implies equation of line is of the form $\log_{10} y = \pm\frac{2}{3}\log_{10} x + 4$ oe

(Eg allow for M1 A0 $\log_{10} y = \pm 0.67\log_{10} x + 4$) Condone $"y" = \pm\frac{2}{3}"x" + 4$ here

A1 $\log_{10} y = -\frac{2}{3}\log_{10} x + 4$ oe such as $3\log_{10} y + 2\log_{10} x = 12$

Allow unsimplified equations such as $\log_{10} y = -\frac{4}{6}\log_{10} x + 4$

This may be implied by the constants if the full equation is used in part (b).
So marks in (a) can be awarded from work in (b).

(b) **Main Method: Starting from** $\log_{10} y = a\log_{10} x + b$ **and working towards** $y = px^q$

M1 Uses one correct log law. Look for either

$$\log_{10} y = a\log_{10} x + b \rightarrow \log_{10} y = \log_{10} x^a + b$$

$$\log_{10} y = a\log_{10} x + b \rightarrow \log_{10} y = a\log_{10} x + \log_{10} 10^b$$

$$\log_{10} y = a\log_{10} x + b \rightarrow y = 10^{a\log_{10} x + b}$$

dM1 A full attempt to get y in terms of x or values of p and q

Cannot be awarded from easier equations, e.g. where $b = 0$

Look for $\log_{10} y = a\log_{10} x + b \rightarrow y = x^a \times 10^b$

A1 $y = 10\,000x^{-\frac{2}{3}}$ but condone $y = 10^4 x^{-\frac{2}{3}}$

This requires the equation and not just the values of p and q

It is acceptable to just write down the answer, but it must follow a correct part (a)

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Method Two: Starting from $y = px^q$ and working towards $\log_{10} y = a\log_{10} x + b$.

M1 Takes $\log_{10}$ of both sides and uses one correct log law

E.g. proceeds to $\log_{10} y = \log_{10} p + \log_{10} x^q$

dM1 Proceeds to $\log_{10} y = \log_{10} p + q\log_{10} x$ and finds p and q using $\log_{10} p = '4'$ and $q = '-\frac{2}{3}'$

A1 $y = 10\,000x^{-\frac{2}{3}}$ or $y = 10^4 x^{-\frac{2}{3}}$

Condone $\log_{10} \leftrightarrow \log$ throughout

This requires the equation and not just the values of p and q

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Alt (b) using the alternative equation in (a)

Method Three: Starting from $a\log_{10} y + b\log_{10} x = c$ and working towards $y = px^q$

M1 Uses correct log laws to combine the two terms on the lhs

Eg: $3\log_{10} y + 2\log_{10} x = 12 \Rightarrow \log_{10} y^3x^2 = 12$

dM1 Undoes the logs and makes y the subject $\Rightarrow y^3x^2 = 10^{12} \Rightarrow y^3x^2 = 10^{12} \Rightarrow y = \sqrt[3]{\frac{10^{12}}{x^2}}$

A1 $y = 10\,000x^{-\frac{2}{3}}$ or $y = 10^4 x^{-\frac{2}{3}}$

……

Method 4: Using the coordinates

M1: Finds either p or q using one of the coordinates.

Look for either

using $(\log_{10} x, \log_{10} y) = (0,4) \Rightarrow x = 1, y = 10000$ and then substituting into $y = px^q \Rightarrow p = 10^4$

Condone slips here for the method mark

Or using $(\log_{10} x, \log_{10} y) = (6,0) \Rightarrow x = 10^6, y = 1$ and the substituting into

$y = 10^4 \times x^q \Rightarrow 1 = 10^4 \times 10^{6q} \Rightarrow q = ...$ Condone slips here for the method mark

dM1 Finds both p and q using both coordinates. (See above)

A1 $y = 10\,000x^{-\frac{2}{3}}$

……

Partially correct answers without any work

If you see any partially correct answers, e.g. $y = 4x^{-\frac{2}{3}}$, without working, award SC 100.

Question Number	Scheme	Marks
4. (i) (a)	$f'(x)=\dfrac{4(x-3)(2x+5)-(2x+5)^2}{(x-3)^2}$ or $\dfrac{(x-3)(8x+20)-(4x^2+20x+25)}{(x-3)^2}$	M1 A1
	$=\dfrac{(2x+5)(2x-17)}{(x-3)^2}$	M1 A1
(b)	Attempts both critical values or finds one "correct" end	M1
	$x<-2.5,\ x>8.5$ (accept $x\leqslant -2.5,\ x\geqslant 8.5$)	A1
		(6)
(ii)	Attempts the chain rule on $(\sin 4x)^{\frac{1}{2}}\to A(\sin 4x)^{-\frac{1}{2}}\times\cos 4x$	M1
	$g(x)=x(\sin 4x)^{\frac{1}{2}}\Rightarrow g'(x)=(\sin 4x)^{\frac{1}{2}}+x\times\frac{1}{2}(\sin 4x)^{-\frac{1}{2}}4\cos 4x$	M1 A1
	Sets $g'(x)=0\to(\sin 4x)^{\frac{1}{2}}+x\times\dfrac{2\cos 4x}{(\sin 4x)^{\frac{1}{2}}}=0$ and $\times\dfrac{(\sin 4x)^{\frac{1}{2}}}{\cos 4x}$ oe	M1
	$\to\tan 4x+2x=0$	A1
		(5) **11 marks**

(i)(a)

M1 Attempts the quotient rule and achieves

$$f'(x)=\frac{A(x-3)(2x+5)-B(2x+5)^2}{(x-3)^2}\quad A,B>0\text{ condoning slips}$$

Alternatively uses the product rule and achieves

$$\frac{d}{dx}\left\{(x-3)^{-1}(2x+5)^2\right\}=\pm A(2x+5)^2(x-3)^{-2}+B(x-3)^{-1}(2x+5)\quad A,B>0$$

They may attempt to multiply out the $(2x+5)^2$ first which is fine as long as they reach a 3TQ.

A1 Score for correct unsimplified $f'(x)$

M1 Attempts to take out a factor of $(2x+5)$ or multiplies out and attempts to factorise the numerator.

The method must be seen $\dfrac{(x-3)4(2x+5)\pm(2x+5)^2}{\ldots}=\dfrac{(2x+5)\{(x-3)4\pm(2x+5)\}}{\ldots}$ condoning slips.

If the method is not seen it may be implied by a correct result for their fraction

This can be achieved from an incorrect quotient or product rule. E.g. $\dfrac{vu'+uv'}{v^2}$ or $\dfrac{vu'-uv'}{v}$

It can be scored by candidates who multiply out their numerators and then factorise by taking out a factor of $(2x+5)$

If the product rule is used it would be for writing as a single fraction and taking out, from the numerator, a common factor of $(2x+5)$.

A1 $\dfrac{(2x+5)(2x-17)}{(x-3)^2}$ but accept expressions such as $\dfrac{4(x+2.5)(x-8.5)}{(x-3)^2}$ or $\dfrac{(2x+5)(2x-17)}{(x-3)(x-3)}$

Note the final two marks in (i)(a) may be scored in (i)(b), ONLY IF the correct work is done on the complete fraction, not just the numerator

(i)(b)

M1 Achieves the two critical values from the quadratic numerator of their $\mathrm{f}'(x)$

Alternatively finds one correct end for their $(2x+5)(2x-17)>0$ or $(2x+5)(2x-17)\geqslant 0$

So award for either $x<-2.5$ or $x>$ their 8.5 which may be scored from an intermediate line.

A1 $x<-2.5, x>8.5$ (accept $x\leqslant -2.5,\ x\geqslant 8.5$).

Ignore any references to "and" or "or" so condone $x<-2.5$ and $x>8.5$

Mark the final response. This is not isw

It may follow working such as $(2x+5)(2x-17)>0 \Rightarrow x>-\frac{5}{2}, x>\frac{17}{2}$. So $x<-2.5, x>8.5$

Accept alternative forms such as $(-\infty,-2.5]\cup[8.5,\infty)$

(ii)

M1 Attempts the chain rule on $(\sin 4x)^{\frac{1}{2}} \to A(\sin 4x)^{-\frac{1}{2}} \times \cos 4x$

M1 For an attempt at the product rule.

If they state $u=x, v=(\sin 4x)^{\frac{1}{2}}, u'=1, v'=\ldots$ award for $(\sin 4x)^{\frac{1}{2}} + x \times$ their v'

If this is not stated or implied by their $uv'+vu'$ then award for $(\sin 4x)^{\frac{1}{2}} + x\times(\sin 4x)^{-\frac{1}{2}}\ldots$

A1 $(\mathrm{g}'(x)) = (\sin 4x)^{\frac{1}{2}} + 2x(\sin 4x)^{-\frac{1}{2}}\cos 4x$ which may be unsimplified.

You may not see the lhs which is fine. Condone $\sin 4x^{\frac{1}{2}}$ for $(\sin 4x)^{\frac{1}{2}}$ if subsequent work is correct

M1 Sets their $\mathrm{g}'(x)$ which must be of the form $(\sin 4x)^{\frac{1}{2}} + kx(\sin 4x)^{-\frac{1}{2}}\cos 4x$ equal to 0

and proceeds with correct work to an equation of the correct form. Allow $\tan 4x = kx$ here

A1 cso $\tan 4x + 2x = 0$

……

Alt to (a) via division which may not be very common

M1 Score for $\frac{(2x+5)^2}{x-3} \to Ax+B+\frac{C}{x-3}$ differentiating to $A \pm \frac{C}{(x-3)^2}$

A1 $4-\frac{121}{(x-3)^2}$

M1 Forms a single fraction and attempts to factorise out $(2x+5)$ from the numerator (which must be a 3TQ)

A1 $\frac{(2x+5)(2x-17)}{(x-3)^2}$

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Alt to (b) via squaring

$$[\mathrm{g}(x)]^2 = x^2\sin 4x \Rightarrow 2\mathrm{g}(x)\mathrm{g}'(x) = 2x\sin 4x + 4x^2\cos 4x$$

M1 Correct form for the rhs. Apply the same rules as the main method. Condone slips on coefficients

dM1 Correct form for the left hand side as well as the right hand side. Condone a slip on the coefficient

A1 $2\mathrm{g}(x)\mathrm{g}'(x) = 2x\sin 4x + 4x^2\cos 4x$

ddM1 Sets $\mathrm{g}'(x)=0$ and proceeds with correct work to an equation of the correct form.

A1 cso $\tan 4x + 2x = 0$

……

Question Number	Scheme	Marks
5 (a)	$12\tan 2x+5\cot x\sec^2 x=0$	
	$12\times\frac{2t}{1-t^2}+5\times\frac{1}{t}\left(1+t^2\right)=0$	B1 M1 A1
	$12\times 2t^2+5\left(1+t^2\right)\left(1-t^2\right)=0\rightarrow 5t^4-24t^2-5=0$ *	A1*
		(4)
(b)	$5t^4-24t^2-5=0$	
	$\left(5t^2+1\right)\left(t^2-5\right)=0$	M1
	Correct order of operations $t=(\pm)\sqrt{5}\Rightarrow x=..$	dM1
	Two of awrt $x=66°,114°,246°,294°$	A1
	All four of awrt $x=65.9°,114.1°,245.9°,294.1°$	A1
		(4) **8 marks**

(a)

B1 Any **correct** identity used within the given equation either in terms of $\tan x$ or in terms of t

Eg: Attempts to replace either $\tan 2x=\frac{2\tan x}{1-\tan^2 x}$, $\cot x=\frac{1}{\tan x}$ or $\sec^2 x=1+t^2$

M1 Uses $\tan 2x=\frac{2\tan x}{1\pm\tan^2 x}$, $\cot x=\frac{1}{\tan x}$, and $\sec^2 x=\pm1\pm\tan^2 x$ or with $t=\tan x$ to produce an equation in terms of t or $\tan x$

A1 Correct intermediate equation in t or $\tan x$ The = 0 may be implied by later work

A1* For proceeding to the correct answer with a correct intermediate line. Must have $=0$.
There cannot be any notational or bracketing errors within the body of the solution if this mark is to be awarded. A notational error is $\tan x^2\leftrightarrow\tan^2 x$
The intermediate line should be of a form in which the given answer could immediately follow. See main scheme for such an example; the fractional terms have been dealt with in this case.
Condone partially completed lines if the candidate is only working on one side of the equation

(b)

M1 Correct attempt to solve.
Allow an attempt to factorise $5t^4-24t^2-5=0\Rightarrow\left(at^2+b\right)\left(ct^2+d\right)=0$ with $ac=\pm5, bd=\pm5$
Alt lets $u=t^2$ and attempts to factorise $5u^2-24u-5=0\Rightarrow$ with usual rules.
Allow use of calculator giving $t^2=5$ or $\tan^2 x=5$ (You may ignore the negative root).
It is also implied by $t=\sqrt{5}$ or $t=-\sqrt{5}$ Watch out for $\tan x=5$ which is M0

dM1 For using the correct order of operations and finding one value of x for their $t^2=k$ where k is a positive constant. Allow accuracy to either the nearest degree or correct to 1dp in radians.
It is dependent upon them having scored the previous M1.

A1 Any two of awrt $x=66°,114°,246°,294°$ May be implied by awrt two of 1.15, 1.99, 4.29, 5.13

A1 All four of awrt $x=65.9°,114.1°,245.9°,294.1°$ AND no extras within the range.

Question Number	Scheme	Marks
6.(a)	$(2.5, 3)$ oe	B1 B1
		(2)
(b)	Attempts one solution usually $4x-10+3=3x-2 \Rightarrow x=5$	M1 A1
	Attempts both solutions $-4x+10+3=3x-2 \Rightarrow x=\frac{15}{7}$	dM1 A1
		(4)
(c)	Attempts to solve $y=kx+2$ with $x=2.5, y=3$ or states that $k<4$	M1
	$k ... \frac{2}{5}$	A1
	States $\frac{2}{5}<k<4$	A1
		(3)
		9 marks

(a)

B1 For one correct coordinate, either $x=2.5$ or $y=3$

B1 For $(2.5, 3)$. Allow $x=..., y=...$ Allow exact equivalents

If a candidate reverses these and just writes down $(3, 2.5)$ score SC 10

(b)

M1 For a correct method of finding one point of intersection. The signs of the terms must be correct
Accept $2(2x-5)+3=3x-2 \Rightarrow x=...$ oe or $-2(2x-5)+3=3x-2 \Rightarrow x=...$ oe
Note that equations such as $2(2x-5)+3=-3x+2 \Rightarrow x=...$ are incorrect and score M0

A1 For 5 or $\frac{15}{7}$ Ignore any reference to the y coordinate

dM1 For a correct method of finding both intersections

A1 For both values 5 and $\frac{15}{7}$ (with no extra solutions)

Ignore any reference to the y coordinates or $x=\frac{2}{3}$ within the body of their solution.

(c)

M1 Attempts to solve $y=kx+2$ with THEIR $x=2.5, y=3$ to find k or deduces that $k<4$
An equivalent is attempting to set k equal to the gradient between (0,2) and (2.5,3)

A1 Finds that $k=\frac{2}{5}$ is a critical value

A1 $\frac{2}{5}<k<4$

……

Alt to (b) via squaring. Cannot be scored via squaring each term.

In reality expect the squaring to start from the point where $2|2x-5| = 3x-2 \pm 3$ is simplified

M1A1 $\quad 4(2x-5)^2 = (3x-5)^2 \Rightarrow 7x^2 - 50x + 75 = 0$

dM1A1 $\quad$ Solve 5 and $\frac{15}{7}$

……

Alternatives to (c)

Alt I

M1A1 Deduces that it must hit the lhs once so $kx+2 = -4x+13 \Rightarrow x = \frac{11}{k+4} < \frac{5}{2} \Rightarrow k > \frac{2}{5}$

Variations on this are

$kx+2 = 4x-7 \Rightarrow x = \frac{9}{4-k} > \frac{5}{2} \Rightarrow k > \frac{2}{5}$ for M1 A1

$\frac{11}{4+k} < \frac{9}{4-k} \Rightarrow k > \frac{2}{5}$ for M1 A1

Alt II via squaring

M1 $\quad$ Sets $2|2x-5| = kx+2 \pm 3$, collects terms, squares and writes in the form $Ax^2 + Bx + C = 0$

FYI they should get $(16-k^2)x^2 + (2k-80)x + 99 = 0$

Then attempts to use $b^2 - 4ac = 0$ with at least *a*, *b* in terms of *k* reaching a value or values for *k*

A1 $\quad k = \frac{2}{5}$

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Question Number	Scheme	Marks
7. (a)	$y\|_{0.8} = 2\cos 2.4 - 2.4 + 4 = 0.13$ AND $y\|_{0.9} = 2\cos 2.7 - 2.7 + 4 = -0.51$	M1
	States change of sign, continuous and hence root	A1
		(2)
(b) (i)	$(x_2) = \frac{1}{3}\arccos(1.5\times 0.8 - 2) = 0.8327$	M1 A1
(ii)	$x_5 = 0.8110$	A1
		(3)
(c)	$\frac{dy}{dx} = -6\sin 3x - 3$	M1 A1
	Attempts $\frac{dy}{dx} = 0 \Rightarrow \sin 3x = -\frac{1}{2} \Rightarrow x = ...$ via correct order of operations	dM1
	Achieves either $\frac{7\pi}{18}$ or $\frac{19\pi}{18}$	A1
	Correct attempt to find the third positive solution e.g. $x = \frac{\frac{\pi}{6} + 3\pi}{3}$	ddM1
	$\beta = \frac{7\pi}{18}$ and $\lambda = \frac{19\pi}{18}$	A1
		(6)
		11 marks

(a)

M1 Attempts the value of y at 0.8 **AND** 0.9 with at least one correct to 1 sf rounded or truncated.
Note that it is possible to choose a tighter interval containing the root but to score the A1 the conclusion must refer to the given interval.

A1 Both values correct to 1sf rounded or truncated, with reason (Sign change **and** continuous function) and minimal conclusion (root)
If the candidate chooses 0.8 and 0.9 the minimal conclusion does not need to mention the interval.
So e.g. $y|_{0.8} = 0.1 > 0,\ y|_{0.9} = -0.5 < 0$ and function is continuous, so ✓ would be acceptable

(b) (i)

M1 Attempts to substitute $x_1 = 0.8$ into the formula. Implied by sight of embedded values in expression or awrt 0.83

A1 AWRT 0.8327

(b)(ii)

A1 $x_5 = 0.8110$ CAO. This is not awrt and 0.811 is A0 unless preceded by 0.8110

If it is clearly marked (b)(ii) then you don't need the x_5

(c)

M1 For $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = A\sin 3x + B$

A1 $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = -6\sin 3x - 3$

dM1 Attempts $\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \sin 3x = a,\quad |a| < 1 \Rightarrow x = ...$ It is dependent upon the previous M

Look for correct order of operations, invsin$a$ then $\div 3$ leading to a value for x.

When $\sin 3x = -\frac{1}{2}$ it is implied, for example, by $-\frac{\pi}{18}, \frac{7\pi}{18}, \frac{11\pi}{18}$ (2nd soln), $\frac{19\pi}{18}$ amongst others or if answers are given as decimals, for example, by awrt -0.17, awrt 1.2 , awrt 1.92 or awrt 3.32

For $\sin 3x = +\frac{1}{2}$ it would be implied, for example, by values such as $\frac{\pi}{18}, \frac{5\pi}{18}$ awrt 0.17 or awrt 0.87

The calculations must be using radians. If degrees are used initially they must be converted to radians

A1 For recognising that either $\frac{7\pi}{18}$ or $\frac{19\pi}{18}$ is a solution to $\sin 3x = -\frac{1}{2}$

ddM1 Attempts to find the solution for λ in the correct quadrant.
Look for the **3rd positive solution** for their $\sin 3x = k$

So for $k > 0$ it would be for $x = \frac{2\pi + \arcsin|k|}{3}$

And for $k < 0$ it would be for $x = \frac{3\pi + \arcsin|k|}{3}$

A1 States that $\beta = \frac{7\pi}{18}$ and $\lambda = \frac{19\pi}{18}$ Labels must be correct

Question Number	Scheme	Marks
8.(i)	$\int \frac{2}{3x-1}\,\mathrm{d}x = \frac{2}{3}\ln\left(3x-1\right)$ $\int_3^{42} \frac{2}{3x-1}\,\mathrm{d}x = \frac{2}{3}\ln\left(125\right) - \frac{2}{3}\ln\left(8\right) = \ln\frac{25}{4}$	M1 A1 dM1 A1 (4)
(ii)	$\frac{2x^3-7x^2+8x+1}{\left(x-1\right)^2} = 2x + B + \frac{C}{\left(x-1\right)^2}$ Full method to find values of A, B and C $\frac{2x^3-7x^2+8x+1}{\left(x-1\right)^2} = 2x - 3 + \frac{4}{\left(x-1\right)^2}$ $\int \mathrm{h}(x)\,\mathrm{d}x = \int Ax + B + \frac{C}{\left(x-1\right)^2}\,\mathrm{d}x$ $= \frac{1}{2}Ax^2 + Bx - \frac{C}{\left(x-1\right)}$ $= x^2 - 3x - \frac{4}{\left(x-1\right)} \quad \left(+c\right)$	B1 M1 A1 M1 A1 ft A1 (6) **10 marks**
(ii) Alt I first 3 marks	$2x^3-7x^2+8x+1 = Ax\left(x-1\right)^2 + B\left(x-1\right)^2 + C$ Any of $A = 2, B = -3$, or $C = 4$ Either substitution or equating coefficients to get two values All values correct $A = 2, B = -3, C = 4$	 B1 M1 A1 **(3)**
(ii) Alt II first 3 marks	$\begin{array}{r} 2x-3 \\ x^2-2x+1\overline{\smash{)}\,2x^3-7x^2+8x+1} \\ \\ \underline{} \\ 4 \end{array}$ Likely to be $2x$ For attempt at division Correct quotient and remainder	 B1 M1 A1 **(3)**

(i)

M1 Integrates to $k\ln(3x-1)$ condoning slips. Condone with a missing bracket.

Please note that $k\ln(3ax-a)$ where a is a positive constant is also correct

If a substitution is made, i.e $u=3x-1$, then they must proceed to $k\ln u$ where k is a constant

A1 $\frac{2}{3}\ln(3x-1)$

Also accept $\frac{2}{3}\ln(3ax-a)$ where a is a positive constant is also correct or $\frac{2}{3}\ln u$ where $u=3x-1$,

Do not allow with the missing bracket unless subsequent work implies that it is present

dM1 Substitutes in both limits and applies one ln law correctly. May be subtraction law or power law.

If a substitution has been made then the correct limits must be used. With u they are 125 and 8

A1 $\ln\frac{25}{4}$ or simplified equivalent such as $\ln 25-\ln 4$, $2\ln 5-2\ln 2$ or $2\ln\frac{5}{2}$

ISW if followed by decimals

(ii)

B1 Any correct value of A, B or C seen or implied.

M1 A full method to find values of A, B and C.

If they attempt $2x^3-7x^2+8x+1=Ax(x-1)^2+B(x-1)^2+C$ via this route, this expression must be correct.

If they attempt by division then they must proceed to a linear quotient but may get a linear remainder.

A1 Correct A, B, C or correct expression. This may be implied by a correct quotient and remainder.

M1 $\int\frac{P}{(x-1)^2}\,\mathrm{d}x\to\frac{Q}{(x-1)^1}$ o.e. where P and Q could be the same

Award for $\int\frac{P}{(x-1)^2}\,\mathrm{d}x\to\frac{Q}{u}$ where they have previously set $u=x-1$

A1ft $\int Ax+B+\frac{C}{(x-1)^2}\,\mathrm{d}x=\frac{1}{2}Ax^2+Bx-\frac{C}{(x-1)}$ or unsimplified equivalent.

So allow $\frac{1}{2}Ax^2+Bx+\frac{C}{-1}(x-1)^{-1}$ with the indices processed

Also allow with non-numerical values.

Also score for $\int Ax+B+\frac{C}{(x-1)^2}\,\mathrm{d}x=\frac{1}{2}Ax^2+Bx-\frac{C}{u}$ where they have previously set $u=x-1$

A1 $x^2-3x-\frac{4}{(x-1)}\quad(+c)$ or exact simplified equivalent with or without the $+c$

So allow $x^2-3x-4(x-1)^{-1}\quad(+c)$

Question Number	Scheme	Marks
9.(a)	$R=\sqrt{41}$	B1
	$\tan\alpha=\frac{4}{5}\Rightarrow\alpha=\text{awrt }0.675$	M1A1
		(3)
(b)	(i) Describes stretch: stretch in the y direction by "$\sqrt{41}$"	B1 ft
	(ii) Describes translation: E.g. translate by $\begin{pmatrix}-\arctan\frac{4}{5}\\0\end{pmatrix}$	B1 ft
		(2)
(c)	Attempts either $g(\theta)=\frac{90}{4+\left(\sqrt{41}\right)^2}$ OR $g(\theta)=\frac{90}{4}$	M1
	Range $2\leqslant g(\theta)\leqslant 22.5$	A1
		(2) **7 marks**

(a)

B1 $R=\sqrt{41}$

Condone $R=\pm\sqrt{41}$ (Do not allow decimals for this mark Eg 6.40 but remember to isw after $\sqrt{41}$)

M1 $\tan\alpha=\pm\frac{4}{5},\ \tan\alpha=\pm\frac{5}{4}\Rightarrow\alpha=\ldots$ Condone $\sin\alpha=4,\ \cos\alpha=5\Rightarrow\tan\alpha=\frac{4}{5}$

If R is used to find α accept $\sin\alpha=\pm\frac{4}{R}$ or $\cos\alpha=\pm\frac{5}{R}\Rightarrow\alpha=\ldots$

A1 $\alpha=\text{awrt }0.675$ Note that the degree equivalent $\alpha=\text{awrt }38.7°$ is A0

(b)(i)

B1ft Fully describes the stretch. Follow through on their R. **Requires the size and the direction**

Allow responses such as

- stretch in the y direction by "$\sqrt{41}$"
- multiplies all the y coordinates/values by "$\sqrt{41}$"
- stretch in ↑ direction by "$\sqrt{41}$"
- vertical stretch by "$\sqrt{41}$"
- Scale Factor "$\sqrt{41}$" in just the y direction

Do **not** award for y is translated/transformed by "$\sqrt{41}$"

(b)(ii)

B1 ft Fully describes the translation. **Requires the size and the direction**

Follow through on their 0.675 or $\alpha =$ awrt 38.7° or $\arctan\frac{4}{5}$

Allow responses such as

- translates left by 0.675
- horizontal by -0.675
- condone "transforms" left by 0.675. (question asks for the translation)
- moves ← by 38.7°
- x values move back by 0.675
- shifts in the negative x direction by $\arctan\frac{4}{5}$
- $\begin{pmatrix}-0.675\\0\end{pmatrix}$

Do **not** award for translates left by -0.675 (double negative...wrong direction)
horizontal shift of 0.675 (no direction)

If there are no labels score in the order given but do allow these to be written in any order as long as the candidate clearly states which one they are answering. For example it is fine to write ….
translation is..............
stretch is ....
If the candidate does not label correctly, or states which one they are doing, but otherwise gets both completely correct then award SC B1 B0

(c)

M1 Score for either end achieved by a correct method

Look for $\frac{90}{4}$ (implied by 22.5) , $\frac{90}{4+\text{their}\left(\sqrt{41}\right)^2}$, $g\dots22.5$ or $g\dots2$ etc

A1 See scheme but allow 22.5 to be written as $\frac{90}{4}$

Accept equivalent ways of writing the interval such as $[2,22.5]$

Condone $2\leqslant \text{g}(x)\leqslant 22.5$ or $2\leqslant y\leqslant 22.5$